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A METHOD OF BOUNDARY PARAMETER ESTIMATION FOR A TWO-DIMENSIONAL DIFFUSION SYSTEM UNDER NOISY OBSERVATIONS

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ABSTRACT

The purpose of this paper is to establish a method for identifying unknown parameters involved in the boundary state of a class of diffusion systems under noisy observations. A mathematical model of the system dynamics is given by a two-dimensional diffusion equation, whose boundary condition is partly unknown due to the existence of an unknown parameter. Noisy observations are made by sensors allocated on the system boundary. Starting with the mathematical model mentioned above, an on-line parameter estimation algorithm is proposed within the framework of the maximum likelihood estimation. Existence of the optimal solution and related necessary conditions are discussed. By solving a local variation of the cost functional with respect to the perturbation of parameters, the estimation mechanism is proposed in a form of recursive computations. Finally, the feasibility of the estimator proposed here is demonstrated through results of digital simulation experiments.

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1. INTRODUCTION

Recently, there has been much interest in the parameter identification problem for distributed parameter systems. For parameter identification problems for a class of distributed parameter systems, excellent survey papers have been published by many researchers (e.g., Kubrusly, 1976). Among them, the method for parameter estimation including discussions on convergence properties of computer algorithm has been developed by Banks and Lamm (1985), Kravaris and Seinfeld (1985), etc. Recently, feasible methods for estimating unknown coefficients which depend on the state and spatial variables were reported by the authors. In this paper, motivated by parameter identification problems of such parameters as heat conduction in chemical reactor or oil reservoir problems (e.g., Chavent, 1978), we deal with the identification of boundary parameters for a two-dimensional diffusion system under noisy observations.

2. PROBLEM CONSIDERED

Let T and G be the time interval $[0,t_f]$ and system domain given by a bounded set in R^2, and let ∂G be the boundary of domain G. The boundary ∂G is characterized by C^2-parameterized curve decomposed into two parts such that

$$\partial G = \partial G_U \cup \partial G_K. \tag{2.1}$$

The system state $u(t,x)$ at time t and at the location $x = (x_1,x_2)$ is governed by heat diffusion equation:

$$\frac{\partial u(t,x)}{\partial t} - \Delta u(t,x) = 0 \quad \text{in } T \times G \tag{2.2a}$$

together with the initial and boundary conditions

$$u(0,x) = u_0(x) \qquad \text{on } G, \tag{2.2b}$$

$$\theta(\xi)\frac{\partial u(t,\xi)}{\partial n} + (1 - \theta(\xi))u(t,\xi) = 0 \quad \text{on } T \times \partial G_U \tag{2.2c}$$

$$\frac{\partial u(t,\xi)}{\partial n} = g(\xi) \qquad \text{on } T \times \partial G_K \tag{2.2d}$$

where $\Delta = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2$, $\partial/\partial n$ denotes the outer normal derivative and $\theta(\xi)$ is unknown parameter with a form of

$$\theta(\xi) = 1/(b(\xi) + 1) \tag{2.3}$$

and where $b(\xi)$ denotes the heat transfer coefficient. Since the heat transfer coefficient b has its physical value of positive definite, then $0 \leq \theta(\xi) \leq 1$. It is apparent that, in the case where $\theta(\xi) \cong 0$, the boundary condition becomes the Dirichlet type while, in the case where $\theta(\xi) \cong 1$, we have the boundary condition of Neumann type.

Observation data are acquired through the sensors located on the boundary part ∂G_K, as shown in Fig. 1. The observation mechanism is modeled by

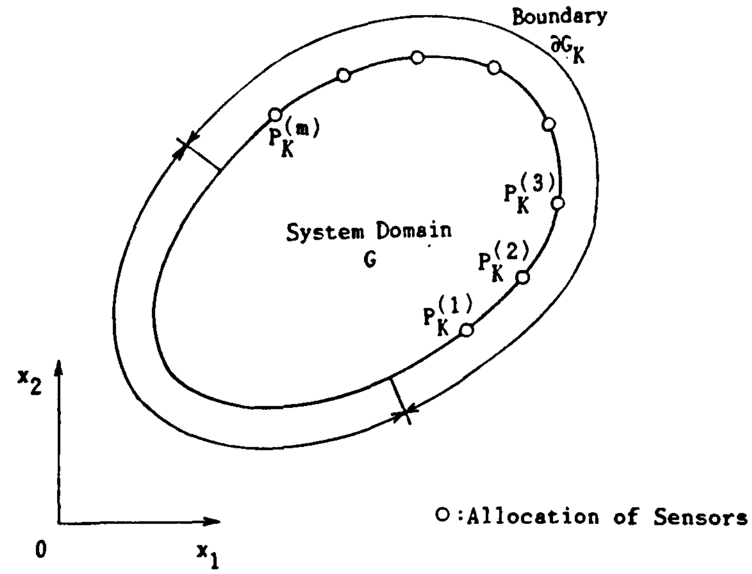

Fig. 1. Location of sensors on the boundary

$$y(t) = \int_0^t H[u(s;\ \theta)]ds + v(t), \tag{2.4}$$

where $y(t) = [y_1(t),\cdots,y_m(t)]'$ and $v(t)$ denotes the observation noise term modeled by an m-dimensional Brownian motion process. In Eq. (2.4), $H[\cdot]$ denotes the signal process defined by

$$\begin{aligned} H[u(t,\theta)] = [h_1u(t,P_K^{(1)};\theta),\cdots \\ \cdots,\ h_mu(t,\ P_K^{(m)};\theta)]' \end{aligned} \tag{2.5}$$

where h_i $(i = 1,\cdots,\ m)$ are assumed to be some constant, and where $P_K^{(i)}$ denotes the coordinates of sensor locations $(z_1^{(i)},z_2^{(i)})$.

The problem considered here is to find a method for identifying parameter $\theta(\xi)$ defined on ∂G_U from information of the a priori boundary known input $g(\xi)$ and the noisy observation data $\{y(t)\}_{t\geq 0}$.

3. THEORETICAL RESULTS FOR PARAMETER ESTIMATION

Let μ_1 and μ_2 be measures induced on the processes $y(t)$ and $\tilde{y}(t) = v(t)$, respectively. Then, the Radon-Nikodym derivative is

$$\frac{d\mu_1}{d\mu_2} = \exp[\int_0^t H[u(s;\theta_o)]'dy(s) - \frac{1}{2}\int_0^t H[u(s;\theta_o)]'H[u(s;\theta_o)]ds] \tag{3.1}$$

where θ_o denotes the true function of unknown parameter. Associated with (3.1), the likelihood ratio functional is given by

$$\Lambda_t(u,y,\theta) = \exp[\int_0^t H[u(s;\theta)]dy(s) - \frac{1}{2}\int_0^t H[u(s;\theta)]'H[u(s;\theta)]ds]. \tag{3.2}$$

The maximum likelihood estimate for the parameter θ is the solution of

$$\inf_{\theta \in H} I_t(\theta) = I_t(\theta^o) \tag{3.3}$$

where Ⓗ denotes the admissible parameter class and

$$I_t(\theta) = -\frac{1}{t}\ln\Lambda_t(u,y,\theta). \tag{3.4}$$

(See Baladrishnan, 1975, for more details.) By using the gradient method (see Polak, 1971), the optimal solution can be obtained by the following recursive computations:

$$\theta^{(i+1)} = \theta^{(i)} + \lambda_i \nabla_\theta I_t(\theta^{(i)}) \tag{3.5a}$$

$$\theta^{(0)} = \overline{\theta} \qquad \text{for } i=1,2,\cdots, \tag{3.5b}$$

where ∇_θ denotes the gradient of the cost $I_t(\theta)$. In order to compute $\nabla_\theta I_t(\theta^{(i)})$, we have to solve the following partial differential equation for each $\theta^{(i)}$:

$$\frac{\partial u(t,x)}{\partial t} - \Delta u(t,x) = 0 \quad \text{in } T \times G \tag{3.6a}$$

with

$$u(0,x) = u_0(x;\theta^{(i)}) \qquad \text{on } G \tag{3.6b}$$

$$\theta^{(i)}(\xi)\frac{\partial u(t,\xi)}{\partial n} + (1-\theta^{(i)}(\xi))u(t,\xi) = 0 \quad \text{on } T \times \partial G_U \tag{3.6c}$$

$$\frac{\partial u(t,\xi)}{\partial n} = g(\xi) \qquad \text{on } T \times \partial G_K. \tag{3.6d}$$

This implies than in order to solve (3.3) massive computations are required. A possible method for avoiding this difficulty is to replace $H[u(t;\theta)]$ in (3.2) by the stationary value $H[\overline{u}(\theta)]$ where $\overline{u}(x;\theta) = \lim_{t\to\infty} u(t,x;\theta)$ and where $\overline{u}(x;\theta)$ is a solution of

$$\Delta\overline{u}(x) = 0 \quad \text{in } G \tag{3.7a}$$

with boundary conditions

$$\theta(\xi)\frac{\partial\bar{u}(\xi)}{\partial n} + (1 - \theta(\xi))\bar{u}(\xi) = 0 \qquad \text{on} \quad \partial G_U \tag{3.7b}$$

$$\frac{\partial\bar{u}(\xi)}{\partial n} = g(\xi) \qquad \text{on} \quad \partial G_K. \tag{3.7c}$$

Thus, we introduce the following cost functional:

$$J_t(\theta) = -\frac{1}{t} H[\bar{u}(\theta)]'y(t) + \frac{1}{2} H[\bar{u}(\theta)]' H[\bar{u}(\theta)]. \tag{3.8}$$

The following result states the relation between the likelihood ratio functional and the cost functional in this paper.

Lemma 1: (See Sunahara, et al., 1984) The cost functional (3.8) satisfies the following asymptotic property:

$$\lim_{t\to\infty} E\{|J_t(\theta) - I_t(\theta)|^2\} = 0. \tag{3.9}$$

Thus, our problem is to find the optimal solution θ_t^* satisfying

$$\inf_{\theta\in(H)} J_t(\theta) = J_t(\theta_t^*). \tag{3.10}$$

In this paper, we shall define the admissible class of parameters as follows:

[Definition] Let Ⓗ be the set of all admissible parameters. Then, Ⓗ is said to be the admissible parameter, i.e., $\theta \in$ Ⓗ, if θ satisfies the following properties:

(P-1) $$0 < \theta(\xi) \leq \beta < 1 \quad \text{on} \quad \partial G_U$$

(P-2) $$\theta \in C^2(\partial G_U).$$

The next results give the existence property for the optimal solution of (3.10).

Theorem 1: Suppose that

(C-1) $$g \in L^2(\partial G_K).$$

Then, there exists at least one solution of (3.10) for a fixed t T with probability one.

The proof of Theorem 1 will be shown in Appendix 1.

The next results show the necessary condition for the optimality of this parameter estimation problem.

Theorem 2: Let θ_t^* be the optimal solution of (3.10). Then the necessary condition for optimality is characterized by the following variational inequality:

$$\int_{\partial G_U} (\theta(\xi) - \theta_t^*(\xi)) \frac{1}{\{\theta_t^*(\xi)-1\}^2}$$

$$\times \frac{\partial \bar{u}(\xi;\theta_t^*)}{\partial n} \frac{\partial p(\xi;\theta_t^*)}{\partial n} d\xi \tag{3.11}$$

$$\text{for } \forall_\theta,\ \theta_t^* \in \Theta$$

where $\bar{u}(\theta_t^*)$ is the solution of (3.7) corresponding to $\theta = \theta_t^*$ and where $p(\theta_t^*)$ is the solution of the following Laplace equation with the nonhomogeneous boundary condition,

$$\Delta p(x) = 0 \quad \text{in } G \tag{3.12a}$$

$$\theta_t^*(\xi)\frac{\partial p(\xi)}{\partial n} + (1 - \theta_t^*(\xi))p(\xi) = 0 \quad \text{on } \partial G_U \tag{3.12b}$$

$$\frac{\partial p(\xi)}{\partial n} = -H^*[\frac{y(t)}{t} - H[\bar{u}(\theta_t^*)]] \quad \text{on } \partial G_K. \tag{3.12c}$$

The proof of Theorem 2 will be shown in Appendix 2.

By using the projection gradient method with the fixed step size λ, the computational algorithm for solving (3.10) can be considered as follows:

$$\theta^{(i+1)} = \Pi_\Theta(\theta^{(i)} - \lambda\nabla_\theta J_t(\theta^{(i)})) \tag{3.13a}$$

$$\theta^{(0)} = \bar{\theta} \qquad \text{for } i = 1,2,\cdots \tag{3.13b}$$

where $\Pi_{\mathbb{H}}$ denotes the projection operator on $\mathbb{H}$. Then, applying Theorem 1, (3.13a) becomes

$$\theta^{(i+1)} = \Pi_{\mathbb{H}}(\theta^{(i)} - \lambda\frac{\partial\bar{u}(\theta^{(i)})}{\partial n}\frac{\partial p(\theta^{(i)})}{\partial n}). \tag{3.13c}$$

In the sequel, we propose the on-line parameter estimator. Let t_0 and t_f be the initial and the terminal time for parameter estimation. We choose the time step k as

$$k = \frac{t_f - t_0}{n} \tag{3.14}$$

and the time interval $[t_0, t_f]$ is divided into

$$\{t_i^{(n)}\}: \quad 0 < t_0 = t_0^{(n)} < t_1^{(n)} < \cdots$$

$$\cdots < t_i^{(n)} < \cdots < t_n^{(n)} = t_f$$

$$t_i = t_0 + ik \qquad \text{for} \quad i = 1,2,\cdots,n-1. \tag{3.15}$$

We compute the recursive algorithm (3.13) at each time $t_i^{(n)}$ by using the observed data $y(t_i^{(n)})$. Considering the fixed step size λ in (3.13) as the time step k, the estimated sequence $\hat{\theta}^{(i)}$ can be obtained at each time $t_i^{(n)}$ for $i = 1,2,\cdots,n$.

Theorem 3: <u>The estimated sequence $\{\hat{\theta}^{(i)}\}_{i=1}^{n}$ is characterized by the following variational inequality:</u>

$$\hat{\theta}^{(i)} \in \mathbb{H} \qquad \text{for } i = 1,2,\cdots,n \tag{3.16a}$$

$$\int_{\partial G_U} (\hat{\theta}^{(i+1)}(\xi) - \hat{\theta}^{(i)}(\xi))$$

$$\times (\phi(\xi) - \hat{\theta}^{(i+1)}(\xi))d\xi$$

$$+ k \int_{\partial G_U} \frac{1}{\{\hat{\theta}^{(i)}(\xi)-1\}^2} \frac{\partial \overline{u}(\xi;\hat{\theta}^{(i)})}{\partial n}$$

$$\times \frac{\partial p(\xi;\hat{\theta}^{(i)})}{\partial n} (\phi(\xi) - \hat{\theta}^{(i+1)}(\xi))d\xi \geq 0$$

$$\text{for } \forall \quad \phi \in \mathbb{H} \qquad i = 1,2,\cdots,n-1 \tag{3.16b}$$

where $\overline{u}$ and p are the solutions of

$$\int_G \overline{u}(x)(\Delta\psi(x))dx = \int_{\partial G_U} \frac{\hat{\theta}^{(i)}(\xi)}{\{\hat{\theta}^{(i)}(\xi)-1\}}$$

$$\times \frac{\partial \overline{u}(\xi)}{\partial n} \frac{\partial \psi(\xi)}{\partial n} d\xi$$

$$- \int_{\partial G_U} \frac{\partial \overline{u}(\xi)}{\partial n} \psi(\xi)d\xi$$

$$+ \int_{\partial G_K} \overline{u}(\xi) \frac{\partial \psi(\xi)}{\partial n} d\xi$$

$$- \int_{\partial G_K} g(\xi)\psi(\xi)d\xi \tag{3.17}$$

$$\int_G p(x)(\Delta\psi(x))dx = \int_{\partial G_U} \frac{\hat{\theta}^{(i)}(\xi)}{\{\hat{\theta}^{(i)}(\xi)-1\}}$$

$$\times \frac{\partial p(\xi)}{\partial n} \frac{\partial \psi(\xi)}{\partial n} d\xi$$

$$- \int_{\partial G_U} \frac{\partial p(\xi)}{\partial n} \psi(\xi) d\xi$$

$$+ \int_{\partial G_K} p(\xi) \frac{\partial \psi(\xi)}{\partial n} d\xi$$

$$+ H[\psi]' \left(\frac{1}{t_i^{(n)}} y(t_i^{(n)}) - H[\bar{u}(\hat{\theta}^{(i)})]\right)$$

$$\text{for } \forall_\psi \in H^2(G).$$

4. ESTIMATION ALGORITHM

4.1. Boundary Element Approach

In order to implement the proposed algorithm in the previous section, we adopt the boundary element method (BEM) (see Brebbia, 1978). Let $v(x,x^i)$ be the fundamental solution satisfying

$$\Delta v(x,x^i) + \delta(x - x^i) = 0, \tag{4.1}$$

where $\delta(x - x^i)$ is the Dirac delta function. It is well-known that $v(x,x^i)$ is given by

$$v(x,x^i) = \frac{1}{2\pi} \ln \frac{1}{r(x,x^i)}, \tag{4.2}$$

where $r(x,x^i)$ denotes the distance between $x = (x_1,x_2)$ and $x^i = (x_1^i,x_2^i)$. By using Green's formula and from (4.2), Eq. (3.7) can be rewritten by the following boundary integral equations:

$$2\pi\ C_i \bar{u}(x^i;\theta)$$

$$+ \int_{\partial G_U} \left\{\frac{\theta(\xi)}{\theta(\xi)-1} \frac{\partial}{\partial n} \left(\ln \frac{1}{r(x^i,\xi)}\right)\right.$$

$$\left. - \ln \frac{1}{r(x^i,\xi)}\right\} \frac{\partial \bar{u}(\xi;\theta)}{\partial n}\, d\xi$$

$$+ \int_{\partial G_K} \frac{\partial}{\partial n} \left(\ln \frac{1}{r(x^i,\xi)}\right) \bar{u}(\xi;\theta)\, d\xi$$

$$= \int_{\partial G_K} \ln \frac{1}{r(x^i,\xi)}\, g(\xi)\, d\xi, \tag{4.3}$$

where C_i denotes the constant such that

$$C_i = \begin{cases} 1 & \text{for } x^i \in G \\ 1/2 & \text{for } x^i \in \partial G \\ 0 & \text{for } x^i \in \overline{G}^c \end{cases} .$$

Similarly, from Eqs. (3.12), we obtain

$$2\pi\ C_i p(x^i;\theta)$$

$$+ \int_{\partial G_U} \left\{\frac{\theta(\xi)}{\theta(\xi)-1} \frac{\partial}{\partial n} \left(\ln \frac{1}{r(x^i,\xi)}\right)\right.$$

$$\left. - \ln \frac{1}{r(x^i,\xi)}\right\} \frac{\partial p(\xi;\theta)}{\partial n}\, d\xi$$

$$+ \int_{\partial G_K} \frac{\partial}{\partial n} \left(\ln \frac{1}{r(x^i,\xi)}\right) p(\xi;\theta)\, d\xi$$

$$= \int_{\partial G_K} \ln \frac{1}{r(x^1,\xi)} H^*[\frac{y(t)}{t} - H[\overline{u}(\theta)]]d\xi . \tag{4.5}$$

In order to perform BEM, the boundary ∂G is approximated by $\partial\tilde{G}$, which is decomposed into $(n + m)$ elements, i.e., as illustrated in Fig. 2

$$\partial\tilde{G} = \partial\tilde{G}_U \cup \partial\tilde{G}_K \tag{4.6a}$$

$$\partial\tilde{G}_U = \partial G_U^{(1)} \cup \cdots \cup \partial G_U^{(n)} \tag{4.6b}$$

$$\partial\tilde{G}_K = \partial G_K^{(1)} \cup \cdots \cup \partial G_K^{(m)} \tag{4.6c}$$

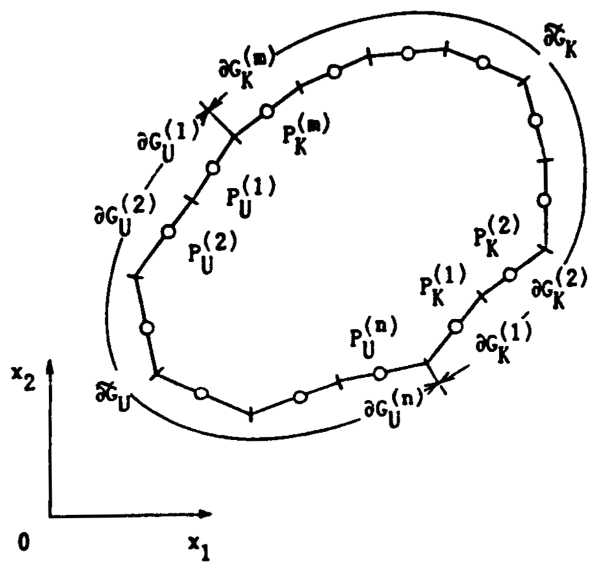

Fig. 2 Boundary elements and nodes of the system.

In Fig. 2, $\{P_U^{(j)}\}_{j=1}^{n}$ and $\{P_K^{(j)}\}_{j=1}^{m}$ denote the node of boundary elements $\{\partial G_U^{(j)}\}_{j=1}^{n}$ and $\{\partial G_K^{(j)}\}_{j=1}^{m}$ respectively. It is noted that the nodes $\{P_K^{(j)}\}_{j=1}^{m}$ coincide with the sensor allocation as shown in Fig. 1. For

For economy of notations, introduce the following vector forms:

$$\{\phi(P_U)\} = [\phi(P_U^{(1)}),\cdots,\phi(P_U^{(n)})]' \tag{4.7a}$$

$$\{\phi(P_K)\} = [\phi(P_K^{(1)}),\cdots,\phi(P_K^{(m)})]'. \tag{4.7b}$$

Then, from (4.4) and (4.5), the boundary steady state is computed by the following vector-matrix equations:

$$\begin{bmatrix} \{\frac{\partial\bar{u}}{\partial n}(P_U;\theta)\} \\ \{\bar{u}(P_K;\theta)\} \end{bmatrix} = A^{-1}(\theta)C \begin{bmatrix} \{0\} \\ \{\bar{g}(P_K)\} \end{bmatrix} \tag{4.8}$$

$$\begin{bmatrix} \{\frac{\partial p}{\partial n}(P_U;\theta)\} \\ \{\bar{p}(P_K;\theta)\} \end{bmatrix} = A^{-1}(\theta)C \begin{bmatrix} \{0\} \\ \{H^*[\frac{y(t)}{t} - H[\bar{u}(\theta)]]\} \end{bmatrix} \tag{4.9}$$

where $A(\theta)$ and C are $(n + m) \times (n + m)$ matrices.

4.2 Estimation Algorithm

The numerical procedure for the estimation algorithm in the previous section can be presented as follows:

Step 0: Select an initial parameter $\hat{\theta}^{(0)}$ at time $t = t_0 > 0$ such that

$$\hat{\theta}^{(0)} = \{\hat{\theta}^{(0)}(P_U)\} \in \Theta,$$

and set $i = 0$.

Step 1: Compute $\{\bar{u}(P_U;\hat{\theta}^{(i)})\}$ and $\{p(P_U;\hat{\theta}^{(i)})\}$ by solving (4.8) and (4.9).

Step 2: Compute the estimated parameter $\hat{\theta}^{(i+1)}$ at time $t = t_0 + (i + 1)k$ by

$$\hat{\theta}^{(i+n)}(P_U^{(j)}) = \hat{\theta}^{(i)}(P_U^{(j)})$$

$$+ \lambda \frac{1}{\{\theta(P_U^{(j)}) - 1^2\}}$$

$$\times \frac{\partial \bar{u}(P_U^{(j)};\theta)}{\partial n} \frac{\partial p(P_U^{(j)};\theta)}{\partial n}\bigg|_{\theta=\hat{\theta}^{(i)}}$$

$$\text{for } P_U^{(j)} \text{ on } \partial G_U \tag{4.11a}$$

and

$$\hat{\theta}^{(i+1)}(P_U^{(j)}) = \Pi_{\textcircled{H}} \cdot \hat{\theta}^{(i+n)}(P_U^{(j)}). \tag{4.11b}$$

Step 3: Setting i by $i + 1$, go back to Step 1.

5. NUMERICAL EXPERIMENTS

Throughout numerical experiments, the system domain is given by a rectangle as illustrated in Fig. 3. Figure 3 shows also that the boundary is divided into 20 elements, while ∂G_U and ∂G_K are decomposed into 5 and 15 elements, i.e., $n = 5$ and $m = 15$, respectively. The boundary input g is set as

$$g(\xi) = 500 \qquad \text{on } \partial G_K. \tag{5.1}$$

Observations are made in the form of

$$y_j^{(i+1)} = y_j^{(i)} + h_j u(ik, P_K^{(j)}; \theta_o)k + \Delta v_j^{(i)}$$

$$y_j^{(0)} = 0 \quad \text{for } j = 1,\cdots,15, \quad i = 0,1,2,\cdots, \tag{5.2a}$$

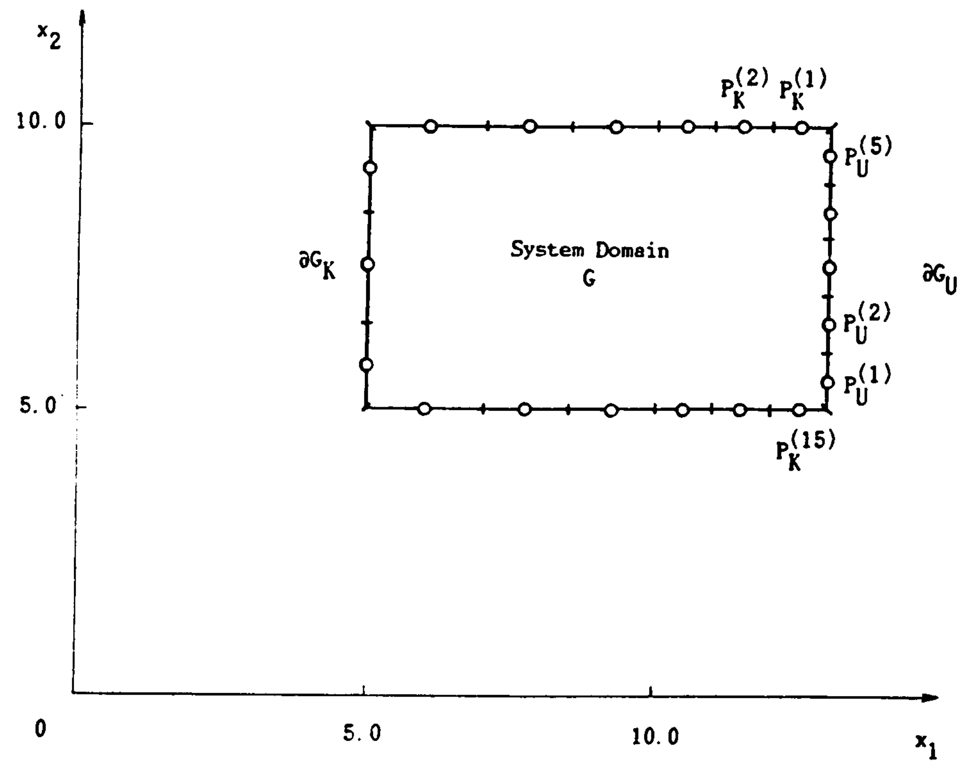

Fig. 3. System domain G and its boundary ∂G

where

$$h_j = 0.1 \quad \text{for} \quad j = 1,\cdots,15, \tag{5.2b}$$

and where $\Delta v_j^{(i)}$ denotes an increment of the standard Brownian motion process generated by the Gaussian distribution. In Eq. (5.2), the system state $u(t,x;\alpha)$ is computed by adopting BEM.

Example 1: The unknown boundary parameter is preassigned as $\theta_o = 0.2$. By substituting known boundary input g and observation data $\{y^{(i)}\}$ $(i = 1,2,\cdots)$ into the proposed algorithm, starting with an initial parameter at time $t_0 = 200 \times k$ $(k = 1.0)$, the estimated parameter $\{\hat{\theta}^{(i)}(\xi)\}_{i=0,1,\cdots}$ could be computed. Results of this example are demonstrated in Fig. 4.

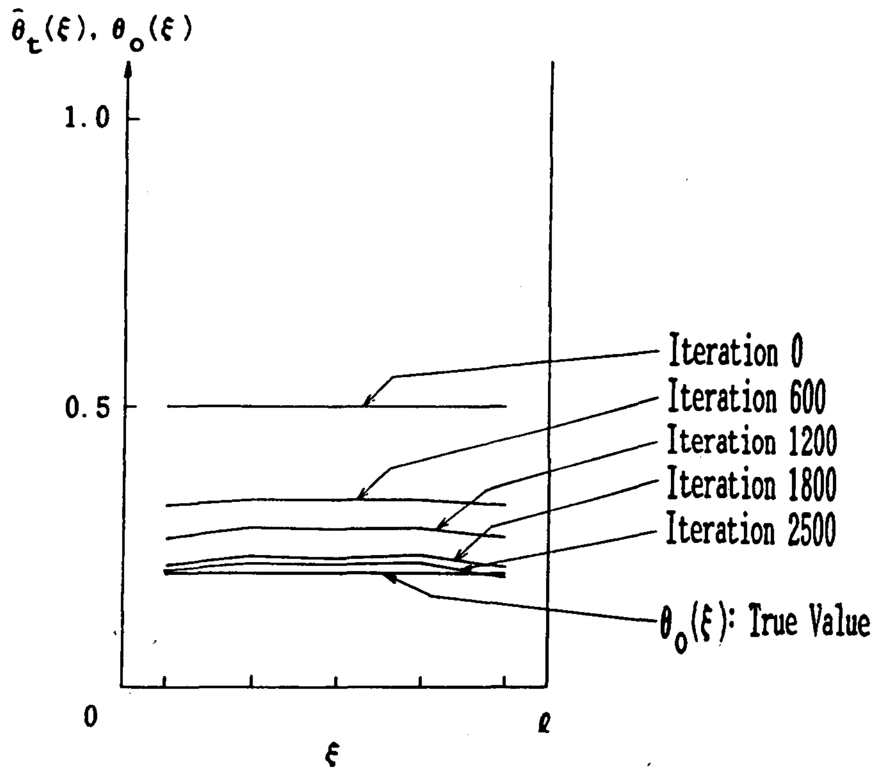

Fig. 4. The estimated parameter in Example 1.

Example 2: In Example 2, we deal with the case where the unknown parameter is a space-variable function. The unknown parameter and simulation results are illustrated in Fig. 5.

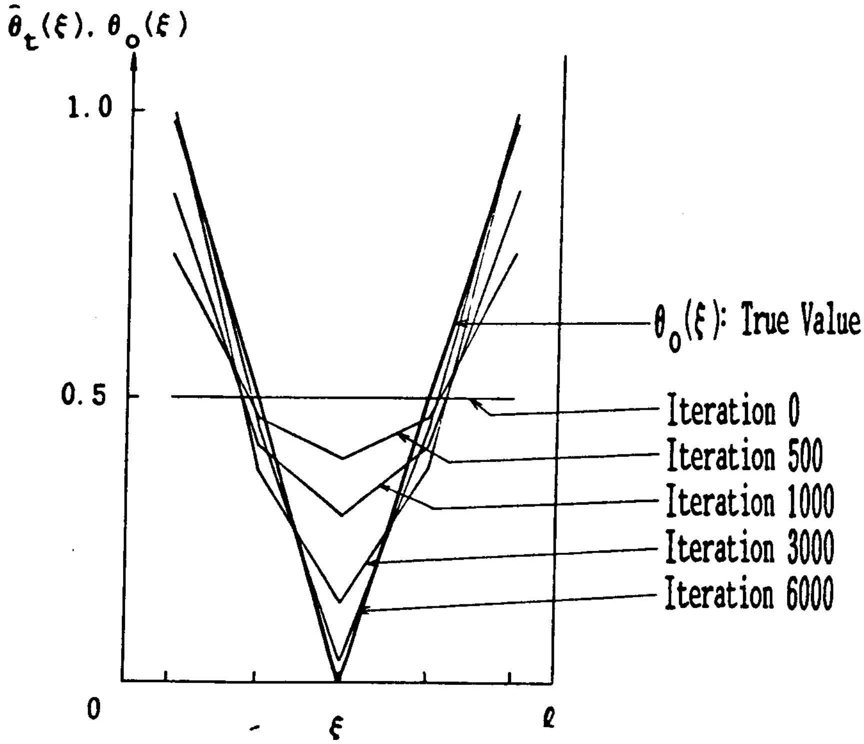

Fig. 5. The estimated parameter in Example 2.

6. CONCLUSIONS

A method for identifying the boundary condition has been developed in this paper for a distributed system whose dynamics are governed by a two-dimensional heat diffusion equation. Based on the concept of maximum likelihood estimate, the problem was converted into the optimal control problem. A difficulty arises in the computational burden involved in computing the gradient of the associated likelihood ratio functional. In this paper, to avoid this difficulty, the steady state model was introduced. The validity of the estimator proposed here was demonstrated through numerical experiments.

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Appendix 1: The Proof of Theorem 1

Let $\{\theta_n\} \in \textcircled{H}$ be a minimizing sequence such that

$$J_t(\theta_n) \to J_t(\theta_t^*). \tag{A.1}$$

From (3.8), it yields that, for $\theta_n, \bar{\theta} \in \textcircled{H}$,

$$\begin{aligned} |J_t(\theta_n) - J_t(\bar{\theta})| \leq (\frac{1}{t} \|y(t)\|_{R^{(m)}} \\ + \frac{1}{2} \|H[\bar{u}(\theta_n) + u(\bar{\theta})]\|_{R^{(m)}}) \\ \times \|H[\bar{u}(\theta_n) - u(\bar{\theta})]\|_{R^{(m)}}. \end{aligned} \tag{A.2}$$

Noting that

$$\|\bar{u}(\theta_n)\|_{H^1(G)} \leq C_1 \tag{A.3}$$

and

$$E\{\frac{1}{t} \|y(t)\|_{R^{(m)}}\} \leq C_2 \quad \text{for a fixed} \quad t \in T, \tag{A.4}$$

we obtain

$$E\{|J_t(\theta_n) - J_t(\bar{\theta})|\} \leq C_3 \|\bar{u}(\theta_n) - u(\bar{\theta})\|_{H^2(G)}. \tag{A.5}$$

Using the compactness method, it can be shown that, under the condition (C-1),

$$\bar{u}(\theta n) \to \bar{u}(\bar{\theta}) \quad \text{strongly in} \quad H^2(G). \tag{A.6}$$

Hence, we have

$$E\{|J_t(\theta_n) - J_t(\overline{\theta})|\} \to 0. \tag{A.7}$$

From (A.1) and (A.5), we may find $\overline{\theta} = \theta_t^*$. The proof has been completed.

Appendix 2. The Proof of Theorem 2

If the optimal solution θ_t^* exists, the following variational inequality holds:

$$\nabla_\theta J_t(\theta_t^*)\cdot(\theta - \theta_t^*) \geq 0 \tag{A.8}$$

where ∇_θ denotes the gradient of $J_t(\theta)$. In order to obtain $\nabla_\theta J_t(\theta)$, let us derive the local variation of cost functional (3.8). Let θ^λ be the perturbed parameter of θ such that

$$\theta^\lambda(\xi) = \theta(\xi) + \lambda\ \delta\theta(\xi) \quad \text{for} \quad \xi \in \partial G_U \tag{A.9}$$

where λ is a positive constant and $\delta\theta(\xi)$ is a given real valued function. Set as

$$\delta J_t(\theta) = J_t(\theta^\lambda) - J_t(\theta), \tag{A.10}$$

and

$$\delta\bar{u}(x;\theta) = \bar{u}(x;\theta^\lambda) - \bar{u}(x;\theta). \tag{A.11}$$

Applying (3.8) to the mean value theorem, (A.10) becomes

$$\delta J_t(\theta) = -H[\delta\bar{u}(\theta)]' \{\frac{y(t)}{t} - H[\bar{u}(\theta)]\}. \tag{A.12}$$

On the other hand, $\delta u(x;\theta)$ is a solution of

$$\delta\bar{u}(x) = 0 \quad \text{in} \quad G \tag{A.13a}$$

with

$$\theta(\xi)\,\frac{\partial(\delta\bar{u}(\xi))}{\partial n} + (1 - \theta(\xi))\delta\bar{u}(\xi))$$

$$= -\frac{\lambda\delta\theta(\xi)}{1 - \theta(\xi) - \lambda\delta\theta(\xi)}\,\frac{\partial\bar{u}(\xi)}{\partial n} \quad \text{on} \quad \partial G_U \tag{A.13b}$$

$$\frac{\partial(\delta\bar{u}(\xi))}{\partial n} = 0 \quad \text{on} \quad \partial G_K. \tag{A.13c}$$

Introduce the so-called "adjoint system" of which state $p(x;\theta)$ is a solution of

$$\Delta p(x) = 0 \quad \text{in} \quad G \tag{A.14a}$$

with

$$\theta(\xi)\,\frac{\partial p(\xi)}{\partial n} - (1 - \theta(\xi))p(\xi) = 0 \quad \text{on} \quad \partial G_U \tag{A.14b}$$

$$\frac{\partial p(\xi)}{\partial n} = H^*[\frac{y(t)}{t} - H[\bar{u}(\theta)]] \quad \text{on} \quad \partial G_K \tag{A.14c}$$

where $H^*[\cdot]$ denotes the adjoint operator of $H[\cdot]$. By virtue of Green's formula, from (A.13) and (A.14), we have

$$H[\delta\bar{u}(\theta)]'\,\{\frac{y(t)}{t} - H[\bar{u}(\theta)]\}$$

$$= \lambda \int_{\partial G_U} \delta\theta(\xi)\,\frac{1}{\{\theta(\xi)-1\}\{\theta(\xi)+\lambda\delta\theta(\xi)-1\}} \tag{A.15}$$

$$\times\,\frac{\partial\bar{u}(\xi;\theta)}{\partial n}\,\frac{\partial p(\xi;\theta)}{\partial n}\,d\xi.$$

From (A.10) and (A.13), $\delta J_t(\theta)$ is computed by

$$\delta J_t(\theta) = -\lambda \int_{\partial G_U} \delta\theta(\xi) \frac{1}{\{\theta(\xi)-1\}\{\theta(\varepsilon)+\lambda\delta\theta(\xi)-1\}} \tag{A.16}$$

$$\times \frac{\partial \bar{u}(\xi;\theta)}{\partial n} \frac{\partial p(\xi;\theta)}{\partial n} d\xi .$$

Thus, we obtain

$$\nabla_\theta J_t(\theta)\cdot\delta\theta = \lim_{\lambda\to 0} \frac{1}{\lambda} \delta J_t(\theta)$$

$$= -\int_{\partial G_U} \delta\theta(\xi) \frac{1}{\{\theta(\xi)-1\}^2} \tag{A.17}$$

$$\times \frac{\partial \bar{u}(\xi;\theta)}{\partial n} \frac{\partial p(\xi;\theta)}{\partial n} d\xi .$$

By setting $\theta = \theta_t^*$ and $\delta\theta = \theta - \theta_t^*$ in (A.17), we can obtain the variational inequality (3.11).

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16. Abstract

The purpose of this paper is to establish a method for identifying unknown parameters involved in the boundary state of a class of diffusion systems under noisy observations. A mathematical model of the system dynamics is given by a two-dimensional diffusion equation, whose boundary condition is partly unknown due to the existence of an unknown parameter. Noisy observations are made by sensors allocated on the system boundary. Starting with the mathematical model mentioned above, an on-line parameter estimation algorithm is proposed within the framework of the maximum likelihood estimation. Existence of the optimal solution and related necessary conditions are discussed. By solving a local variation of the cost functional with respect to the perturbation of parameters, the estimation mechanism is proposed in a form of recursive computations. Finally, the feasibility of the estimator proposed here is demonstrated through results of digital simulation experiments.

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